



Georg Cantor.

THE  
*Mystery*  
OF THE  
*Aleph*

*Mathematics, the Kabbalah,  
and the Search for Infinity*

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LA VILLETTE  
MÉDIATHÈQUE

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*To Miriam, who at age 6 understood something about the  
difference between  $\aleph_0$  and the power of the continuum.*

No

Halle

On January 6, 1918, an emaciated and weary man died of heart failure at the Halle Nervenlinik, a university mental clinic in the German industrial city of Halle. His body was quietly transported across town for burial in a small cemetery. Only a few people attended the Lutheran ceremony, including the man's widow and five surviving children.

The cemetery no longer exists; it has since been razed to make ground for private homes. But someone saved the headstone, and years later it was relocated, without the body, to another small burial ground in Halle, where it can still be seen. The chiseled inscription reads:

Dr. Georg Cantor  
Professor d. Mathematik  
3.3.1845-6.1.1918

By the time of his death, Georg Cantor had been hospitalized at the Halle Nervenlinik for seven months. But this stay at the clinic was not his first. Georg Cantor had been admitted,

released, and readmitted to the clinic many times. And his mental problems had begun years before the clinic was built in 1891.

Georg Cantor received his doctorate in mathematics in 1869 from the University of Berlin, where he had studied under some of the world's greatest mathematicians and absorbed many important ideas in mathematics. He was eager to put his knowledge to use developing new theories in the field of mathematical analysis. The twenty-four-year-old was excited at the prospect of obtaining his first teaching position at a German university, hoping it would allow him time to pursue his research. But upon graduation, Cantor's only offer was from Friedrich's University in Halle, some seventy miles southwest of Berlin.

Halle is an old city with charming medieval cobblestoned streets. It was founded in the middle of the tenth century as a center of salt production on the Saale River. The city survived the bombings of the world war, and many ancient buildings still stand in the historic city center, where people stroll to shops and cafés unimpeded by motorized traffic. Halle is called the City of Five Towers. The four spires of the medieval Marktkirche loom over the lower buildings in the center of town, and nearby stands the fifth, the Red Tower of 1418, a monument to the struggle of the townspeople for independence from an oppressive aristocracy.

In 1685, the composer Georg Friedrich Handel was born in Halle in a house whose oldest standing walls date back to the twelfth century. Handel lived in this house for 18 years. The house, now a museum dedicated to the composer's life, can still be visited. Halle has always been a city of concerts, opera, and music of the people.



Marketplace of Halle an der Saale about 1900.

Credit: Fritz Mueller, Courtesy of H. H. H. H.

By all rights, Halle should have held some attraction for Cantor, as his family members on both sides were gifted musicians. Some of them had achieved renown in their native Russia. But Cantor was not interested in the charms of Halle. His was a family of immigrants—from the Iberian Peninsula via Denmark and Russia—and young Cantor was pushed to excel. His father, in particular, sent Georg letters throughout the years urging him to do well at school and to live up to the great expectations of his family.

Halle is situated halfway between two great university cities: Berlin to the northeast and Göttingen to the west. During the late nineteenth century, the University of Berlin was the world's best in mathematics, and Berlin was one of the most vibrant and exciting cities in all Europe. Göttingen was the other academic magnet. Like Halle, Göttingen is an old medieval city. Many houses in the town center bear plaques with the names of famous former residents, from Heine the poet to Bunsen the chemist to Olbers the astronomer, and many others, most notable among them Carl Friedrich Gauss (1777-1855), arguably the greatest mathematician of the time. Cantor felt the pull of both Berlin and Göttingen.

But Cantor stayed in Halle, waiting for the invitation that never came. Over the years, whenever a mathematics opening became available at Berlin or Göttingen he pinned his hopes on it, and when he wasn't offered the position, he would go into a fit of rage. He had an intense, demanding personality, and an explosive nature. These attributes made him enemies and lost him friends throughout his life. In contrast with his behavior with other mathematicians, Cantor exhibited tenderness in his relationships with his family mem-

bers. While he always dominated conversations with colleagues, at home Cantor took a more relaxed role, letting his wife and children initiate and lead conversations at the dinner table. He ended every meal by asking his wife: "Have you been pleased with me today, and do you love me?"

Cantor started as a *Privatdozent*, the entry-level academic job at German universities of the time. Within a few years of hard work, he was promoted to Associate Professor, and shortly afterwards a Professor of Mathematics. Cantor became involved in intensive research in mathematics, but in the midst of his most productive period, something strange happened, which put a temporary end to his work. In the summer of 1884, Georg Cantor was struck by deep depression. From May through June of that year he was immobilized—unable to work or do much of anything. His condition distressed his wife and children and perplexed his colleagues, who saw in him a mathematician aspiring to great heights. However, without any professional help or medication, Cantor recovered from his illness and returned to normal life. Afterwards, he wrote a letter to a close friend, the Swedish mathematician Gösta Mittag-Leffler (1846-1927), describing his illness and mentioning that just before the mental breakdown he was working on the "continuum problem."

The following year, 1885, Cantor built an opulent house for his family on Handelstrasse, a street named after Halle's great composer. The house is still owned by Cantor's grandson. It is a two-story building, with high ceilings and tall windows. Georg Cantor's father, a merchant and stockbroker, had died a few years earlier, leaving his heirs half a million marks. Some of the inheritance money went into building the



new house and buying furnishings so that the Cantor family could live in comfort. Today as then, Handelstrasse is a quiet, tree-lined street on which there are many expensive homes, and the house is ten minutes' walk from the university and from cafés, restaurants, and cultural institutions. But Cantor did not stay home with his family long enough to enjoy the new house. Shortly he fell ill again. This time, too, just before his mental breakdown, Cantor had been working on the continuum problem.

The University of Halle had an excellent department of psychiatry. Cantor could get the best treatment available at the time—and it was free, since he was a university professor. His university and the Ministry of Culture in Berlin, which authorized all such decisions, were generous in granting Cantor repeated leaves of absence from his teaching duties. But his hospitalizations became more frequent as the years went by. In the Prussian State Archives in Berlin there is a letter on budgets sent by the Culture Ministry to the Finance Ministry, dated August 29, 1902. In this letter, the Minister of Culture requests, among other things, an appropriation of 6,660 German marks to support a substitute appointment of Professor of Mathematics at the University of Halle in case Professor Dr. Cantor should be too ill to resume his duties. But Cantor recovered yet again and returned to teaching.

Within a year he was ill again, and was readmitted to the Nervenlinik on September 17, 1904, remaining there until March 1, 1905. Then in the fall of that year, Cantor was back in the clinic.

The Halle Nervenlinik is a complex of eleven buildings



Cantor's house.

*Credit: Amir D. Aczel.*

constructed of attractive yellow-glazed bricks situated within a large fenced compound. The quality of construction was so high that the facility looks today almost exactly as it did when it was built over a century ago. The main building, with its pointed tower, resembles a military headquarters rather than a mental clinic. Inside, the rooms are spacious with large windows and private baths. This was not a place where people were restrained with straitjackets. It was—and still is—a clinic for short stays of several months by wealthy individuals whose families could afford room and board and treatment. Georg Cantor, a professor at the university, was given a single room with good view and had the freedom to pursue his research. His treatment consisted mainly of periods of soaking in a hot bath.

And although he did die while hospitalized at the clinic, there was certainly no justification for the statement Bertrand Russell later made about Cantor (referring to a letter Cantor had written) that those who will read his letter will not be surprised to hear that he died in an insane asylum.

We don't know the precise nature of Cantor's illness. Some of his reported symptoms resemble those associated with bipolar disorder, or manic depression. But the causes of this mental illness are now generally attributed to genetic factors, and in Cantor's ancestry there are no known cases of the disease.

One fact is known about Georg Cantor's illness. His attacks of depression were all associated with periods in which he was thinking about what is now known as "Cantor's continuum hypothesis." He was contemplating a single mathematical expression, an equation using the Hebrew letter aleph:

$$2^{\aleph_0} = \aleph_1$$

This equation is a statement about the nature of infinity. A century and a third after Cantor first wrote it down, the equation—along with its properties and implications—remains the most enduring mystery in mathematics.